

MATHEMATICS

Chapter 6: The Triangle and Its Properties



THE TRIANGLE AND ITS PROPERTIES

A triangle is a simple closed curve made up of three line segments.

Classification of triangles:

On the basis of sides: A triangle is said to be

- an isosceles triangle if two of its sides are equal;
- a scalene triangle if all of its three sides are of different lengths;
- an equilateral triangle if all of its sides are of equal lengths.

On the basis of angles: A triangle is said to be

- an acute triangle if each one of its angles measure less than 90° ;
- an obtuse triangle if one of its angles measure more than 90° ;
- a right triangle if one of its angles measure 90° .

Angle sum property of a triangle: The sum of all three interior angles of a triangle is 180° .

In an equilateral triangle:

- All sides have same length.
- each angle has measure 60° .

In an isosceles triangle:

- two sides have the same length.
- base angles opposite to equal sides are equal.

1. The sum of any two sides of a triangle is always greater than the third side.
2. The difference of any two sides of a triangle is always less than the third side.
3. An exterior angle of a triangle is formed when a side of a triangle is produced. There are two ways of forming an exterior angle at each vertex of a triangle.
4. Exterior angle property of a triangle: The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles.
5. In a right angled triangle, the side opposite to the right angle is called the hypotenuse and the other two sides are called its legs.
6. Pythagoras property: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides.
7. A median of a triangle is the line segment joining a vertex of the triangle to the mid-point of its opposite side. A triangle has three medians.
8. An altitude of triangle is the perpendicular drawn from a vertex of a triangle to its opposite sides. A triangle has three altitudes.

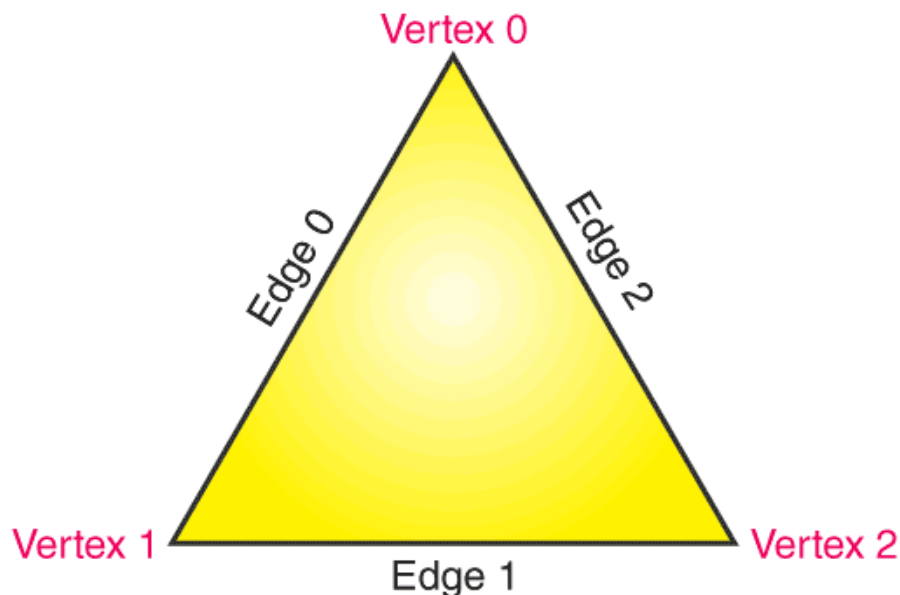
Triangle

A triangle is a closed curve made of three line segments.

In Geometry, a triangle is a three-sided polygon that consists of three edges and three

vertices. The most important property of a triangle is that the sum of the internal angles of a triangle is equal to 180 degrees. This property is called angle sum property of triangle.

If ABC is a triangle, then it is denoted as $\triangle ABC$, where A, B and C are the vertices of the triangle. A triangle is a two-dimensional shape, in Euclidean geometry, which is seen as three non-collinear points in a unique plane.



Shape of Triangle

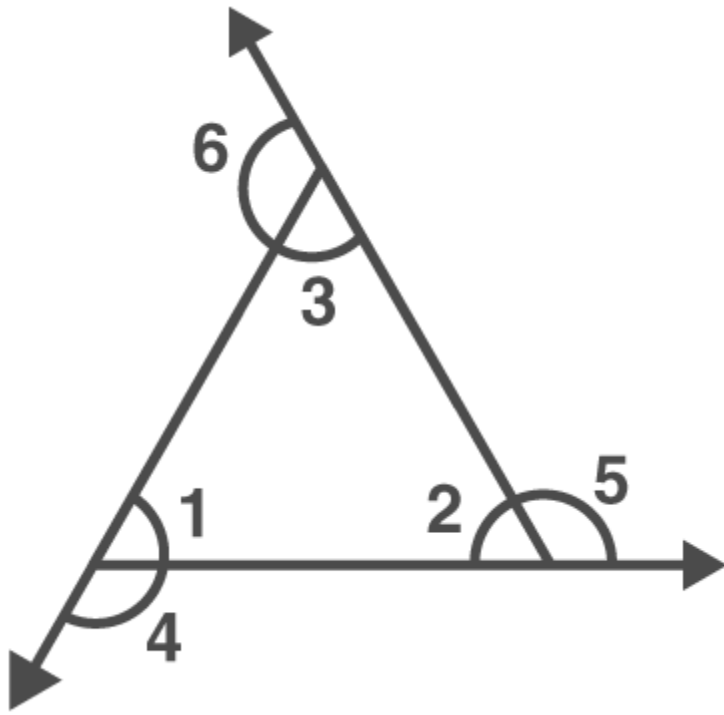
Triangle is a closed two-dimensional shape. It is a three-sided polygon. All sides are made of straight lines. The point where two straight lines join is the vertex. Hence, the triangle has three vertices. Each vertex forms an angle.

Angles of Triangle

There are three angles in a triangle. These angles are formed by two sides of the triangle, which meets at a common point, known as the vertex. The sum of all three interior angles is equal to 180 degrees.

If we extend the side length outwards, then it forms an exterior angle. The sum of consecutive interior and exterior angles of a triangle is supplementary.

Let us say, $\angle 1$, $\angle 2$ and $\angle 3$ are the interior angles of a triangle. When we extend the sides of the triangle in the outward direction, then the three exterior angles formed are $\angle 4$, $\angle 5$ and $\angle 6$, which are consecutive to $\angle 1$, $\angle 2$ and $\angle 3$, respectively.



Hence,

$$\angle 1 + \angle 4 = 180^\circ \text{(i)}$$

$$\angle 2 + \angle 5 = 180^\circ \text{(ii)}$$

$$\angle 3 + \angle 6 = 180^\circ \text{(iii)}$$

If we add the above three equations, we get;

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 180^\circ + 180^\circ + 180^\circ$$

Now, by angle sum property we know,

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

Therefore,

$$180 + \angle 4 + \angle 5 + \angle 6 = 180^\circ + 180^\circ + 180^\circ$$

$$\angle 4 + \angle 5 + \angle 6 = 360^\circ$$

This proves that the sum of the exterior angles of a triangle is equal to 360 degrees.

Properties

Each and every shape in Maths has some properties which distinguish them from each other. Let us discuss here some of the properties of triangles.

- A triangle has three sides and three angles.
- The sum of the angles of a triangle is always 180 degrees.
- The exterior angles of a triangle always add up to 360 degrees.
- The sum of consecutive interior and exterior angle is supplementary.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Similarly, the difference between the lengths of any two sides of a triangle

is less than the length of the third side.

- The shortest side is always opposite the smallest interior angle. Similarly, the longest side is always opposite the largest interior angle.

Triangle

It has three: Sides:

- (i) Sides: \overline{AB} , \overline{BC} and \overline{CA}
- (ii) Angles: $\angle BAC$, $\angle ACB$ and $\angle CBA$
- (iii) Vertices: A, B and C

Types of Triangle

On the basis of length of the sides, triangles are classified into three categories:

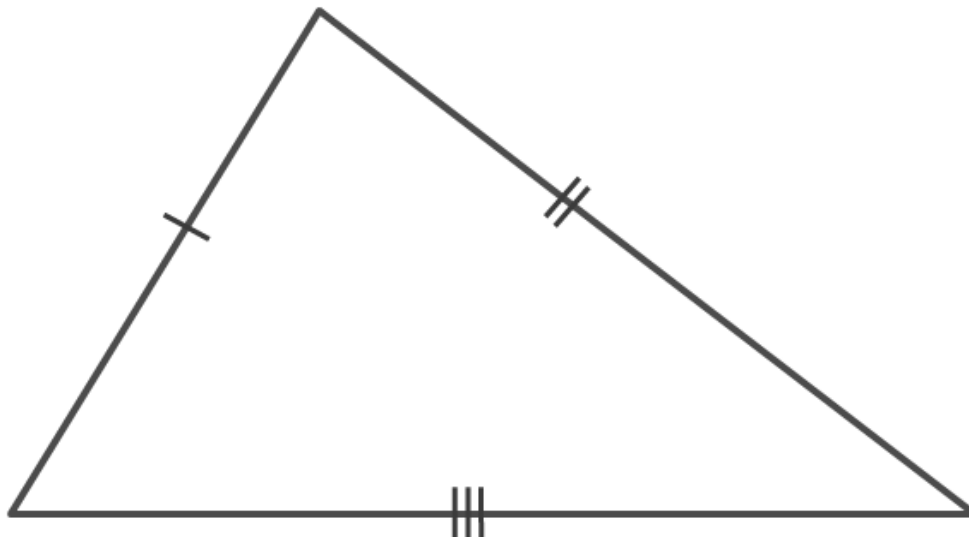
- Scalene Triangle
- Isosceles Triangle
- Equilateral Triangle

On the basis of measurement of the angles, triangles are classified into three categories:

- Acute Angle Triangle
- Right Angle Triangle
- Obtuse Angle Triangle

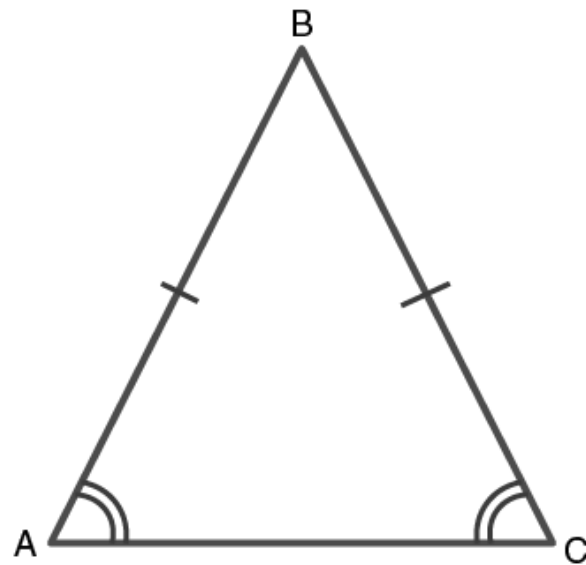
Scalene Triangle

A scalene triangle is a type of triangle, in which all the three sides have different side measures. Due to this, the three angles are also different from each other.



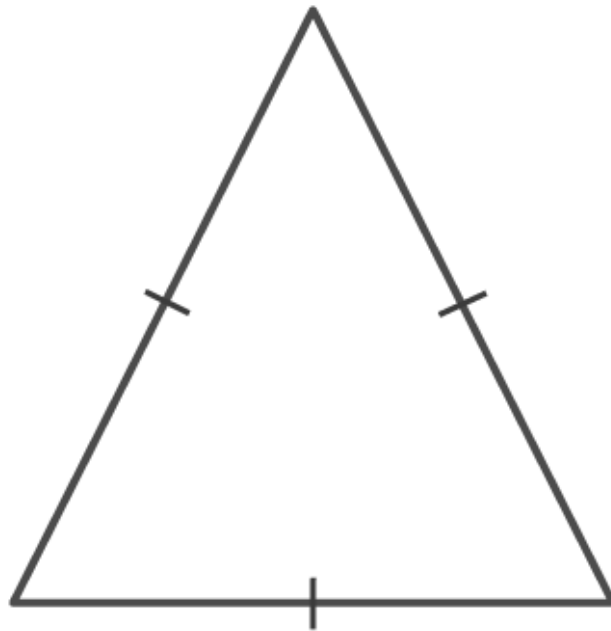
Isosceles Triangle

In an isosceles triangle, two sides have equal length. The two angles opposite to the two equal sides are also equal to each other.



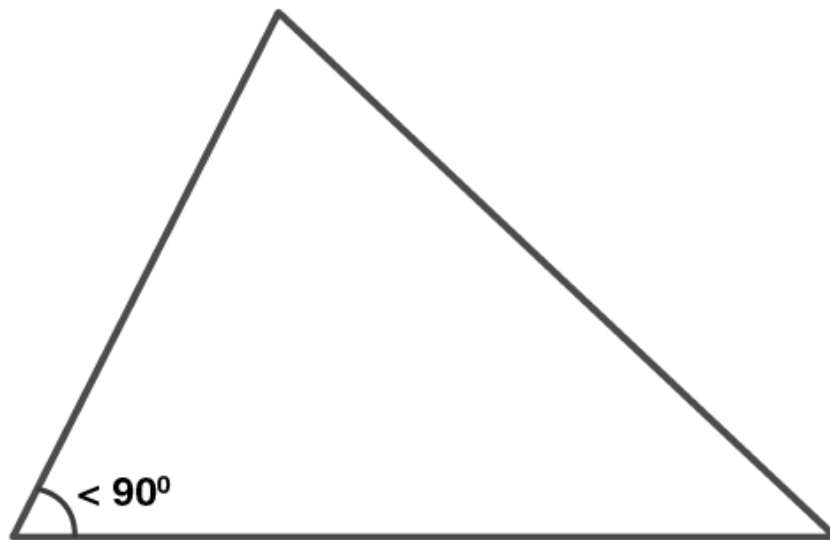
Equilateral Triangle

An equilateral triangle has all three sides equal to each other. Due to this all the internal angles are of equal degrees, i.e. each of the angles is 60°

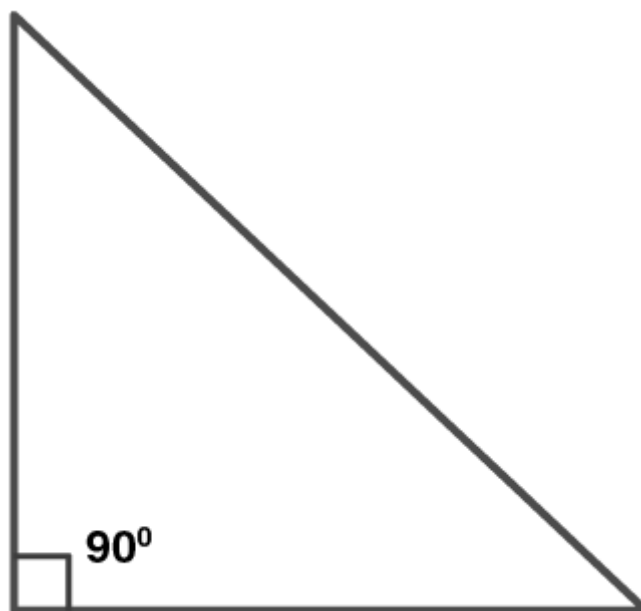


Acute Angled Triangle

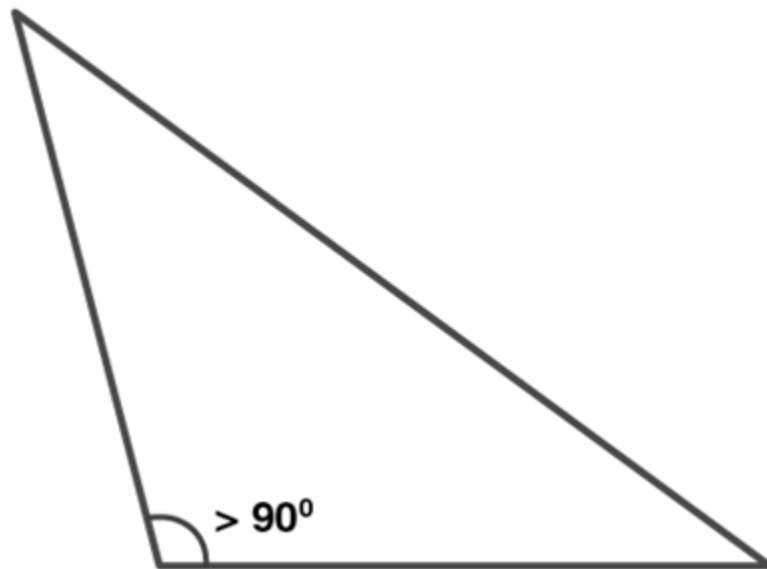
An acute triangle has all of its angles less than 90° .

**Right Angled Triangle**

In a right triangle, one of the angles is equal to 90° or right angle.

**Obtuse Angled Triangle**

An obtuse triangle has any of its one angles more than 90° .

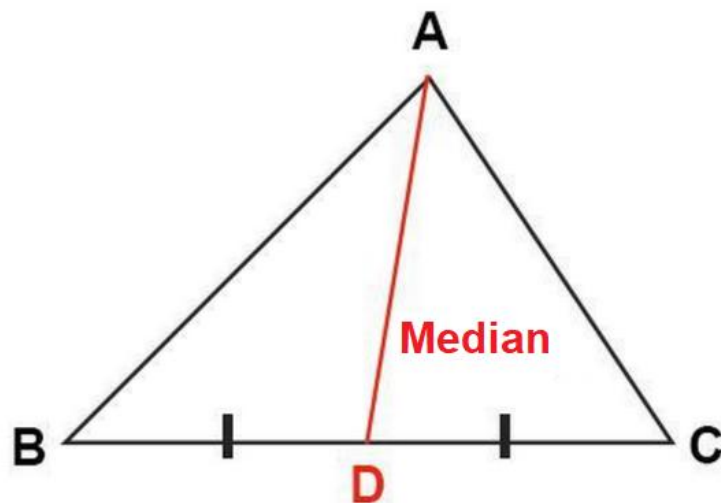


Important Lines in a Triangle

Median

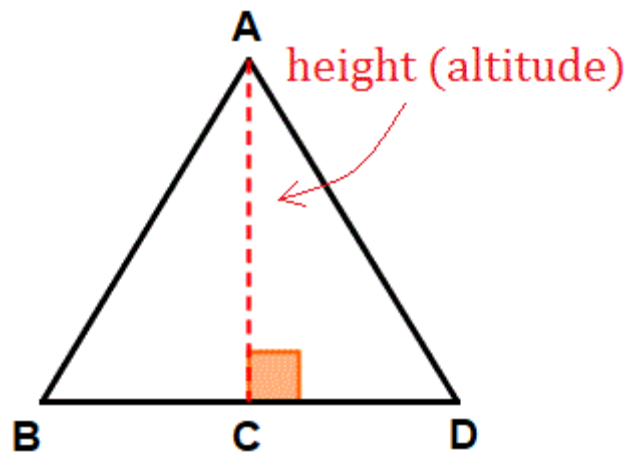
Median is the line that connects a vertex of a triangle to the mid-point of the opposite side.

In the given figure, \overline{AD} is the median, joining the vertex A to the midpoint of \overline{BC} .



Altitude

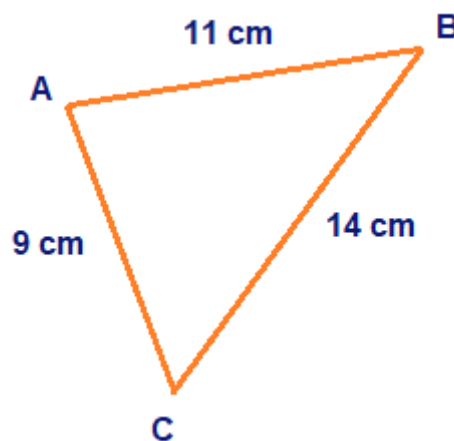
An altitude is a line segment through a vertex of the triangle and perpendicular to a line containing the opposite side.



Sides Also Have Constraints

Sum of the lengths of two sides of a triangle

The sum of the lengths of any two sides of a triangle is greater than the third side.



In the above triangle,

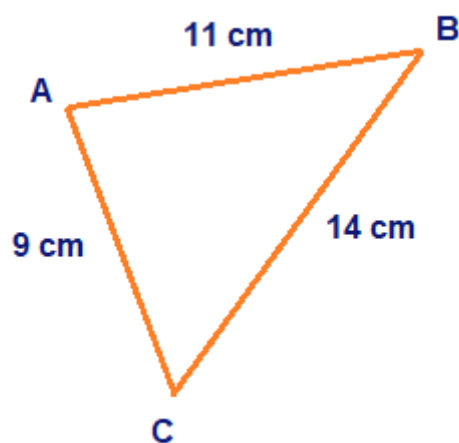
$$9 + 11 = 20 > 14$$

$$11 + 14 = 25 > 9$$

$$9 + 14 = 23 > 11$$

Difference between lengths of two sides of a triangle

The difference between lengths of any two sides is smaller than the length of the third side.



In the above triangle,

$$11 - 9 = 2 < 14$$

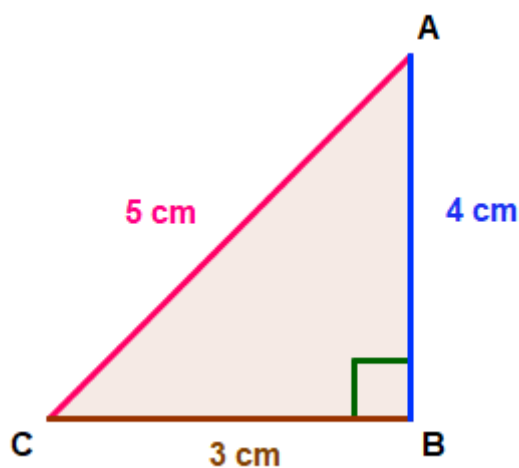
$$14 - 11 = 3 < 9$$

$$14 - 9 = 5 < 11$$

Using the concept of sum of two sides and difference of two sides, it is possible to determine the range of lengths that the third side can take.

Pythagoras Theorem

- The side opposite to the right angle in a right-angled triangle is called the hypotenuse.
- The other two sides are known as legs of the right-angled triangle.
- In a right-angled triangle, square of hypotenuse is equal to the sum of squares of legs.



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 5^2 = 4^2 + 3^2$$

If a triangle holds Pythagoras property, then it is a right-angled triangle.

Properties of isosceles and equilateral triangles

Properties of Isosceles Triangle

Two sides are equal in length.

Base angles opposite to the equal sides are equal.

Properties of Equilateral Triangle

All three sides are equal in length.

Each angle equals to 60° .

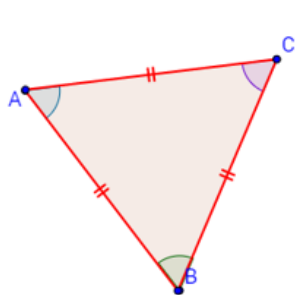
Classification of Triangles

Classification of triangles based on sides

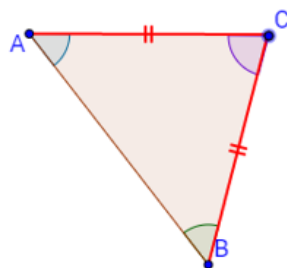
Equilateral triangle: A triangle in which all the three sides are of equal lengths.

Isosceles triangle: A triangle in which two sides are of equal lengths.

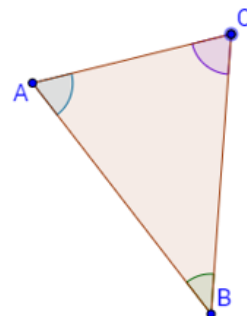
Scalene Triangle: A triangle in which all three sides are of different length.



Equilateral Triangle
All sides same



Isosceles Triangle
Two sides same



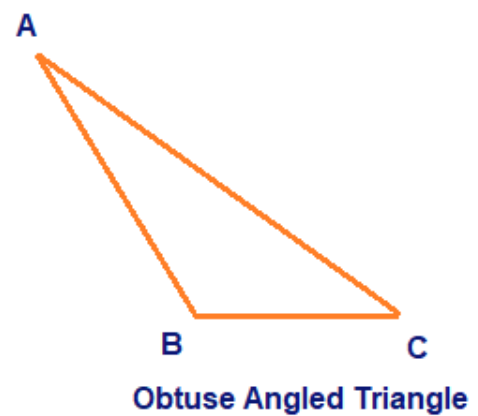
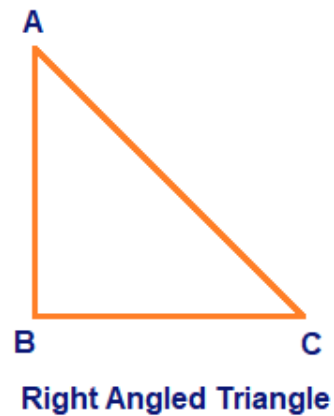
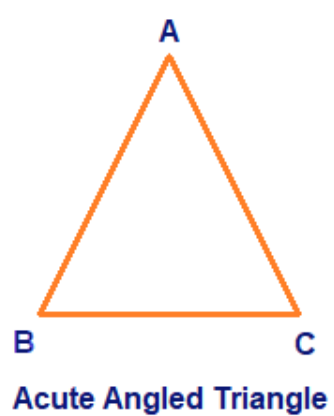
Scalene Triangle
All sides different

Classification of triangles based on angles

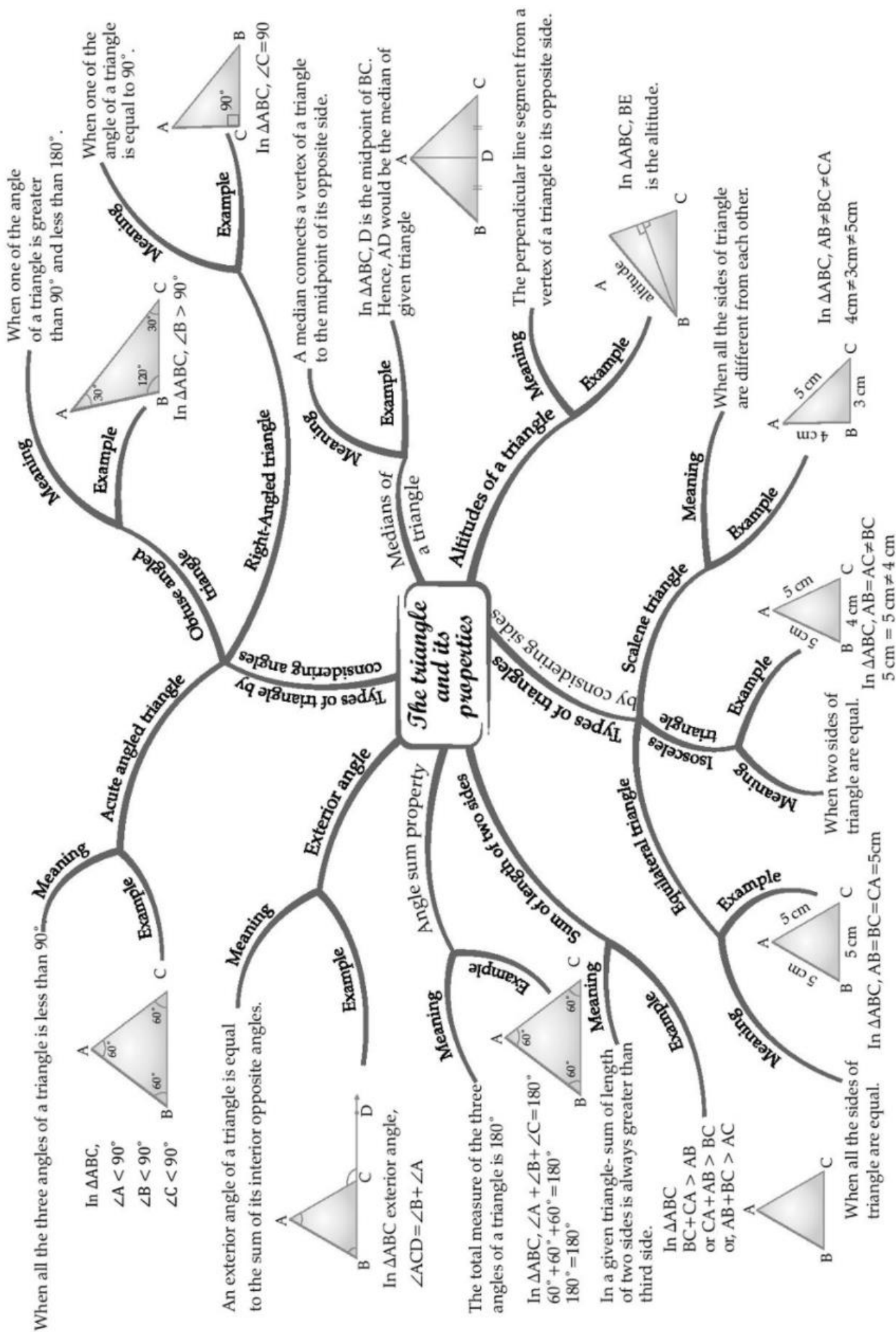
Acute-angled: A triangle with three acute angles.

Right-angled: A triangle with one right angle.

Obtuse-angled: A triangle with one obtuse angle.



CHAPTER - 6 THE TRIANGLE AND ITS PROPERTIES



Important Questions

Multiple Choice Questions :

Question 1. A triangle has how many sides :

- (a) three
- (b) five
- (c) four
- (d) None of these

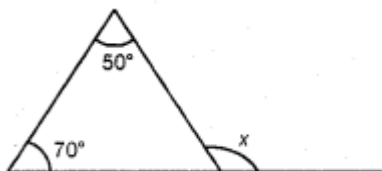
Question 2. A triangle has medians :

- (a) 2
- (b) 1
- (c) 3
- (d) None of these

Question 3. A triangle has altitudes :

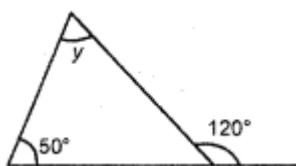
- (a) 2
- (b) 3
- (c) 1
- (d) None of these

Question 4. Find the value of x :



- (a) 120°
- (b) 110°
- (c) 100°
- (d) None of these

Question 5. Find the value of y :



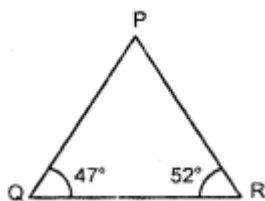
- (a) 50°
- (b) 70°

- (c) 40°
- (d) None of these

Question 6. Sum of three angles of a triangle is :

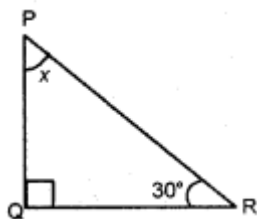
- (a) 170°
- (b) 90°
- (c) 180°
- (d) None of these

Question 7. Find the third angle of the given triangle



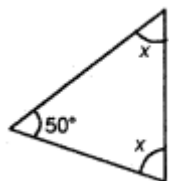
- (a) 71°
- (b) 61°
- (c) 81°
- (d) None of these

Question 8. Find the unknown x in the following diagram



- (a) 60°
- (b) 30°
- (c) 90°
- (d) None of these

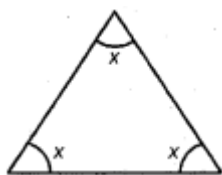
Question 9. Find the value of x in the given diagram :



- (a) 65°
- (b) 50°
- (c) 70°

(d) None of these

Question 10. Find the value of x in the given diagram :



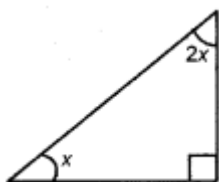
(a) 4°

(b) 60°

(c) 80°

(d) None of these

Question 11. Find the value of x in the given diagram :



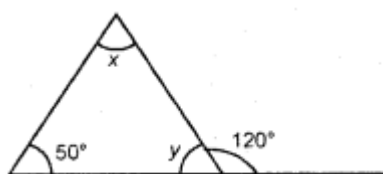
(a) 30°

(b) 45°

(c) 60°

(d) None of these

Question 12. Find the value of x and y in the following diagram



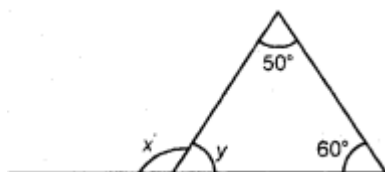
(a) (60, 70)

(b) (50, 70)

(c) (70, 60)

(d) None of these

Question 13. Find the value of x and y in the following diagram



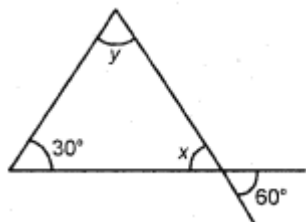
(a) (110, 70)

(b) 70, 110)

(c) (60, 120)

(d) none of these

Question 14. Find the value of x and y in the following diagram :



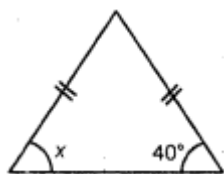
(a) (60,90)

(b) (90,60)

(c) (60,60)

(d) none of these

Question 15. Find the angle x in given diagram :



(a) 30°

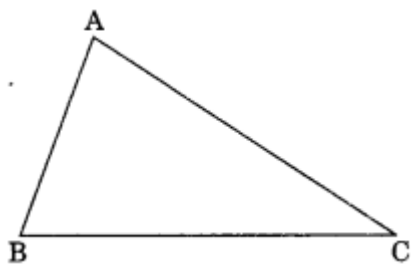
(b) 60°

(c) 40°

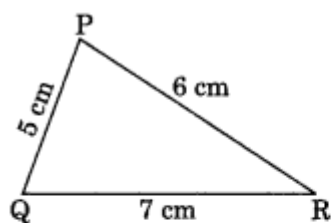
(d) none of these

Very Short Questions :

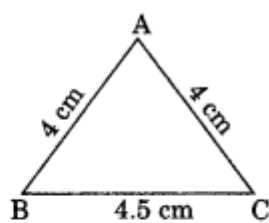
1. In $\triangle ABC$, write the following:
 - (a) Angle opposite to side BC.
 - (b) The side opposite to $\angle ABC$.
 - (c) Vertex opposite to side AC.



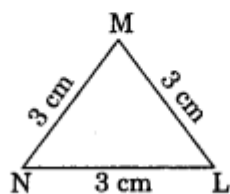
2. Classify the following triangle on the bases of sides



(i)



(ii)

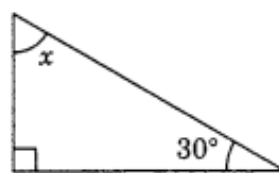


(iii)

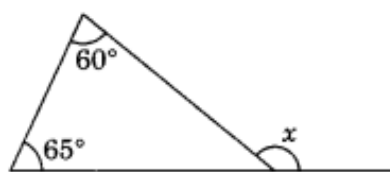
3. In the given figure, name the median and the altitude. Here E is the midpoint of BC.
4. In the given diagrams, find the value of x in each case.



(i)

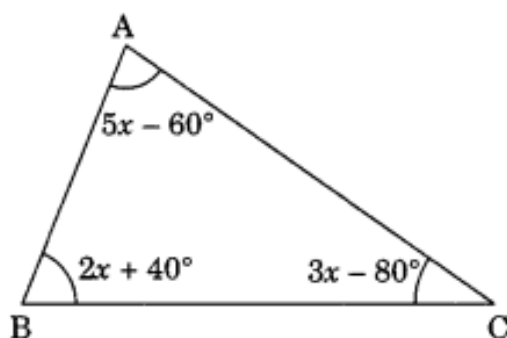


(ii)



(iii)

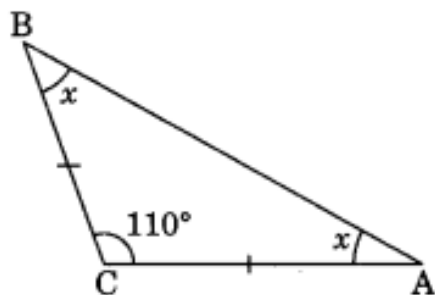
5. Which of the following cannot be the sides of a triangle?
 - (i) 4.5 cm, 3.5 cm, 6.4 cm
 - (ii) 2.5 cm, 3.5 cm, 6.0 cm
 - (iii) 2.5 cm, 4.2 cm, 8 cm
6. In the given figure, find x .



7. One of the equal angles of an isosceles triangle is 50° . Find all the angles

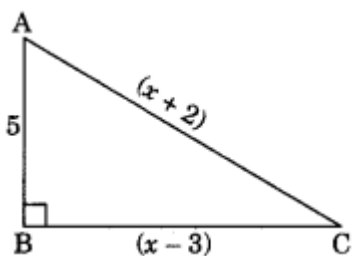
of this triangle.

8. In $\triangle ABC$, $AC = BC$ and $\angle C = 110^\circ$. Find $\angle A$ and $\angle B$.

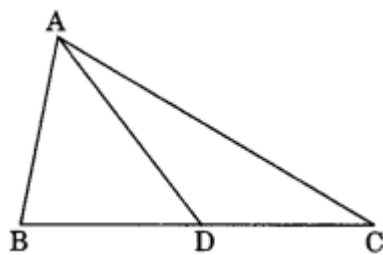


Short Questions :

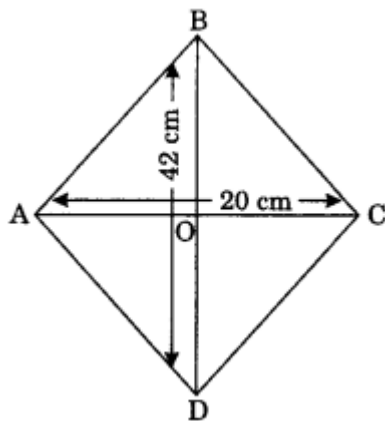
- Two sides of a triangle are 4 cm and 7 cm. What can be the length of its third side to make the triangle possible?
- Find whether the following triplets are Pythagorean or not?
 - (5, 8, 17)
 - (8, 15, 17)
- In the given right-angled triangle ABC, $\angle B = 90^\circ$. Find the value of x .



4. AD is the median of a $\triangle ABC$, prove that $AB + BC + CA > 2AD$ (HOTS)



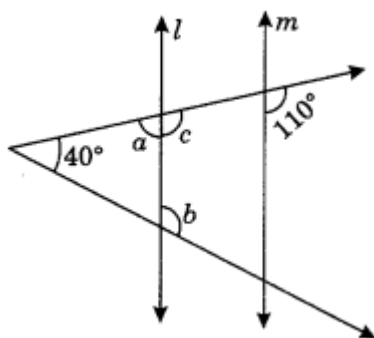
5. The length of the diagonals of a rhombus is 42 cm and 40 cm. Find the perimeter of the rhombus.



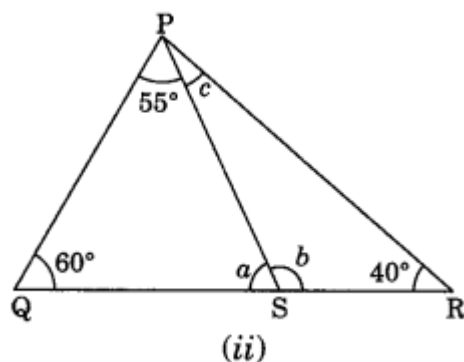
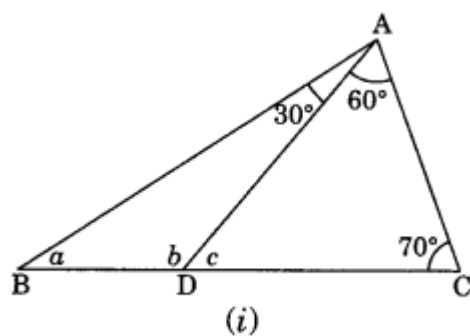
6. The sides of a triangle are in the ratio 3 : 4 : 5. State whether the triangle is right-angled or not.
7. A plane flies 320 km due west and then 240 km due north. Find the shortest distance covered by the plane to reach its original position.

Long Questions :

1. In the following figure, find the unknown angles a and b , if $l \parallel m$.



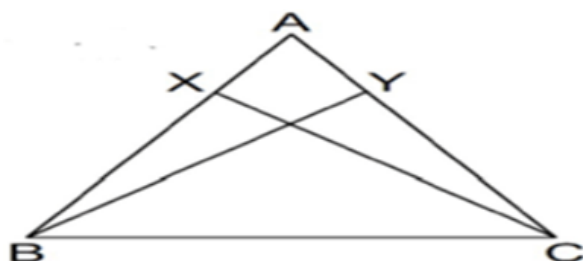
2. In figure (i) and (ii), Find the values of a , b and c .



3. I have three sides. One of my angle measure 15° . Another has a measure of 60° . What kind of a polygon am I? If I am a triangle, then what kind of triangle am I?
4. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall
5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Assertion and Reason Questions :

1) Assertion : In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of $\triangle ABC$ such that $AX = AY$ then $CX = BY$



Reason: If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent

- a) both Assertion and reason are correct and reason is correct explanation for Assertion.
- b) both Assertion and reason are correct but reason is not correct explanation for Assertion
- c) Assertion is true but reason is false.
- d) both Assertion and reason are false

2) Assertion: Two angles measures $a - 60^\circ$ and $123^\circ - 2a$. If each one is opposite to equal sides of an isosceles triangle, then the value of a is 61° .

Reason: Sides opposite to equal angles of a triangle are equal.

- a) both Assertion and reason are correct and reason is correct explanation for Assertion.
- b) both Assertion and reason are correct but reason is not correct explanation for Assertion
- c) Assertion is true but reason is false.
- d) both Assertion and reason are false

ANSWER KEY -

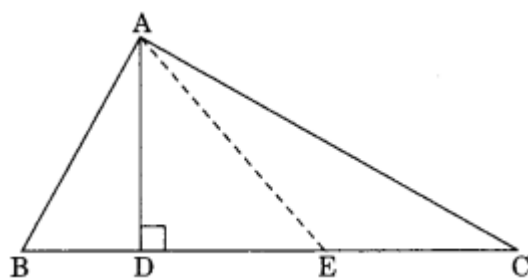
Multiple Choice questions :

1. (a) three

2. (c) 3
3. (b) 3
4. (a) 120°
5. (b) 70°
6. (c) 180°
7. (c) 81°
8. (a) 60°
9. (a) 65°
10. (b) 60°
11. (a) 30°
12. (c) (70, 60)
13. (a) (110, 70)
14. (a) (60, 90)
15. (c) 40°

Very Short Answer :

1. (a) In $\triangle ABC$, Angle opposite to BC is $\angle BAC$
 (b) Side opposite to $\angle ABC$ is AC
 (c) Vertex opposite to side AC is B
2. (i) $PQ = 5$ cm, $PR = 6$ cm and $QR = 7$ cm
 $PQ \neq PR \neq QR$
 Thus, $\triangle PQR$ is a scalene triangle.
 (ii) $AB = 4$ cm, $AC = 4$ cm
 $AB = AC$
 Thus, $\triangle ABC$ is an isosceles triangle.
 (iii) $MN = 3$ cm, $ML = 3$ cm and $NL = 3$ cm
 $MN = ML = NL$
 Thus, $\triangle MNL$ is an equilateral triangle.
3. In $\triangle ABC$, we have



AD is the altitude.

AE is the median.

4. (i) $x + 45^\circ + 30^\circ = 180^\circ$ (Angle sum property of a triangle)

$$\Rightarrow x + 75^\circ = 180^\circ$$

$$\Rightarrow x = 180^\circ - 75^\circ$$

$$x = 105^\circ$$

(ii) Here, the given triangle is right angled triangle.

$$x + 30^\circ = 90^\circ$$

$$\Rightarrow x = 90^\circ - 30^\circ = 60^\circ$$

(iii) $x = 60^\circ + 65^\circ$ (Exterior angle of a triangle is equal to the sum of interior opposite angles)

$$\Rightarrow x = 125^\circ$$

5. (i) Given sides are, 4.5 cm, 3.5 cm, 6.4 cm

$$\text{Sum of any two sides} = 4.5 \text{ cm} + 3.5 \text{ cm} = 8 \text{ cm}$$

Since $8 \text{ cm} > 6.4 \text{ cm}$ (Triangle inequality)

The given sides form a triangle.

(ii) Given sides are 2.5 cm, 3.5 cm, 6.0 cm

$$\text{Sum of any two sides} = 2.5 \text{ cm} + 3.5 \text{ cm} = 6.0 \text{ cm}$$

$$\text{Since } 6.0 \text{ cm} = 6.0 \text{ cm}$$

The given sides do not form a triangle.

(iii) 2.5 cm, 4.2 cm, 8 cm

$$\text{Sum of any two sides} = 2.5 \text{ cm} + 4.2 \text{ cm} = 6.7 \text{ cm}$$

$$\text{Since } 6.7 \text{ cm} < 8 \text{ cm}$$

The given sides do not form a triangle.

6. In $\triangle ABC$, we have

$$5x - 60^\circ + 2x + 40^\circ + 3x - 80^\circ = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$\Rightarrow 5x + 2x + 3x - 60^\circ + 40^\circ - 80^\circ = 180^\circ$$

$$\Rightarrow 10x - 100^\circ = 180^\circ$$

$$\Rightarrow 10x = 180^\circ + 100^\circ$$

$$\Rightarrow 10x = 280^\circ$$

$$\Rightarrow x = 28^\circ$$

Thus, $x = 28^\circ$

7. Let the third angle be x° .

$$x + 50^\circ + 50^\circ = 180^\circ$$

$$\Rightarrow x^\circ + 100^\circ = 180^\circ$$

$$\Rightarrow x^\circ = 180^\circ - 100^\circ = 80^\circ$$

Thus $\angle x = 80^\circ$

In given $\triangle ABC$, $\angle C = 110^\circ$

8. Let $\angle A = \angle B = x^\circ$ (Angle opposite to equal sides of a triangle are equal)

$$x + x + 110^\circ = 180^\circ$$

$$\Rightarrow 2x + 110^\circ = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 110^\circ$$

$$\Rightarrow 2x = 70^\circ$$

$$\Rightarrow x = 35^\circ$$

Thus, $\angle A = \angle B = 35^\circ$

Short Answer :

1. Let the length of the third side be x cm.

Condition I: Sum of two sides $>$ the third side

$$\text{i.e. } 4 + 7 > x \Rightarrow 11 > x \Rightarrow x < 11$$

Condition II: The difference of two sides less than the third side.

$$\text{i.e. } 7 - 4 < x \Rightarrow 3 < x \Rightarrow x > 3$$

Hence the possible value of x are $3 < x < 11$

$$\text{i.e. } x < 3 < 11$$

2. (a) Given triplet: (5, 8, 17)

$$17^2 = 289$$

$$8^2 = 64$$

$$5^2 = 25$$

$$8^2 + 5^2 = 64 + 25 = 89$$

Since $89 \neq 289$

$$5^2 + 8^2 \neq 17^2$$

Hence (5, 8, 17) is not Pythagorean triplet.

(b) Given triplet: (8, 15, 17)

$$17^2 = 289$$

$$15^2 = 225$$

$$8^2 = 64$$

$$15^2 + 8^2 = 225 + 64 = 289$$

$$17^2 = 15^2 + 8^2$$

Hence (8, 15, 17) is a Pythagorean triplet.

3. In $\triangle ABC$, $\angle B = 90^\circ$

$AB^2 + BC^2 = AC^2$ (By Pythagoras property)

$$(5)^2 + (x - 3)^2 = (x + 2)^2$$

$$\Rightarrow 25 + x^2 + 9 - 6x = x^2 + 4 + 4x$$

$$\Rightarrow -6x - 4x = 4 - 9 - 25$$

$$\Rightarrow -10x = -30$$

$$\Rightarrow x = 3$$

Hence, the required value of $x = 3$

4. In $\triangle ABD$,

$$AB + BD > AD \dots(i)$$

(Sum of two sides of a triangle is greater than the third side)

Similarly, In $\triangle ADC$, we have

$$AC + DC > AD \dots(ii)$$

Adding (i) and (ii), we have

$$AB + BD + AC + DC > 2AD$$

$$\Rightarrow AB + (BD + DC) + AC > 2AD$$

$$\Rightarrow AB + BC + AC > 2AD$$

Hence, proved.

5. AC and BD are the diagonals of a rhombus ABCD.

Since the diagonals of a rhombus bisect at the right angle.

$$AC = 40 \text{ cm}$$

$$AO = \frac{40}{2} = 20 \text{ cm}$$

$$BD = 42 \text{ cm}$$

$$OB = \frac{42}{2} = 21 \text{ cm}$$

In right angled triangle AOB, we have

$$AO^2 + OB^2 = AB^2$$

$$\Rightarrow 20^2 + 21^2 = AB^2$$

$$\Rightarrow 400 + 441 = AB^2$$

$$\Rightarrow 841 = AB^2$$

$$\Rightarrow AB = \sqrt{841} = 29 \text{ cm.}$$

Perimeter of the rhombus = $4 \times \text{side} = 4 \times 29 = 116 \text{ cm}$

Hence, the required perimeter = 116 cm

6. Let the sides of the given triangle are $3x$, $4x$ and $5x$ units.

For right angled triangle, we have

Square of the longer side = Sum of the square of the other two sides

$$(5x)^2 = (3x)^2 + (4x)^2$$

$$\Rightarrow 25x^2 = 9x^2 + 16x^2$$

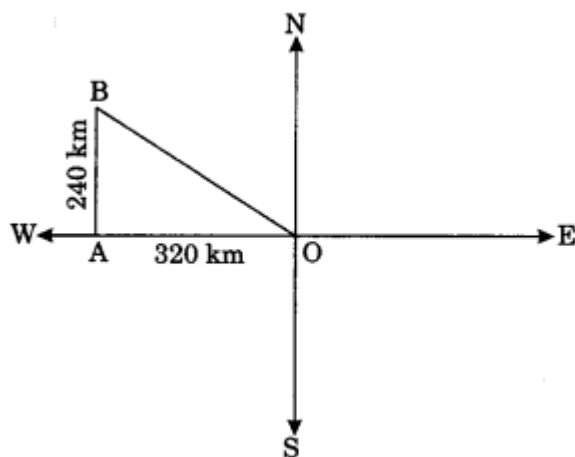
$$\Rightarrow 25x^2 = 25x^2$$

Hence, the given triangle is a right-angled.

Here, $OA = 320 \text{ km}$

$AB = 240 \text{ km}$

$OB = ?$



Clearly, ΔOBA is right angled triangle

$OB^2 = OA^2 + AB^2$ (By Pythagoras property)

$$\Rightarrow OB^2 = 320^2 + 240^2$$

$$\Rightarrow OB^2 = 102400 + 57600$$

$$\Rightarrow OB^2 = 160000$$

$$\Rightarrow OB = \sqrt{160000} = 400 \text{ km.}$$

Hence the required shortest distance = 400 km.

Long Answer :

1. Here, $l \parallel m$

$$\angle c = 110^\circ \text{ (Corresponding angles)}$$

$$\angle c + \angle a = 180^\circ \text{ (Linear pair)}$$

$$\Rightarrow 110^\circ + \angle a = 180^\circ$$

$$\Rightarrow \angle a = 180^\circ - 110^\circ = 70^\circ$$

$$\text{Now } \angle b = 40^\circ + \angle a \text{ (Exterior angle of a triangle)}$$

$$\Rightarrow \angle b = 40^\circ + 70^\circ = 110^\circ$$

Hence, the values of unknown angles are $a = 70^\circ$ and $b = 110^\circ$

2. (i) In $\triangle ADC$, we have

$$\angle c + 60^\circ + 70^\circ = 180^\circ \text{ (Angle sum property)}$$

$$\Rightarrow \angle c + 130^\circ = 180^\circ$$

$$\Rightarrow \angle c = 180^\circ - 130^\circ = 50^\circ$$

$$\angle c + \angle b = 180^\circ \text{ (Linear pair)}$$

$$\Rightarrow 50^\circ + \angle b = 180^\circ$$

$$\Rightarrow \angle b = 180^\circ - 50^\circ = 130^\circ$$

In $\triangle ABD$, we have

$$\angle a + \angle b + 30^\circ = 180^\circ \text{ (Angle sum property)}$$

$$\Rightarrow \angle a + 130^\circ + 30^\circ = 180^\circ$$

$$\Rightarrow \angle a + 160^\circ = 180^\circ$$

$$\Rightarrow \angle a = 180^\circ - 160^\circ = 20^\circ$$

Hence, the required values are $a = 20^\circ$, $b = 130^\circ$ and $c = 50^\circ$

(ii) In $\triangle PQS$, we have

$$\angle a + 60^\circ + 55^\circ = 180^\circ \text{ (Angle sum property)}$$

$$\Rightarrow \angle a + 115^\circ = 180^\circ$$

$$\Rightarrow \angle a = 180^\circ - 115^\circ$$

$$\Rightarrow \angle a = 65^\circ$$

$$\angle a + \angle b = 180^\circ \text{ (Linear pair)}$$

$$\Rightarrow 65^\circ + \angle b = 180^\circ$$

$$\Rightarrow \angle b = 180^\circ - 65^\circ = 115^\circ$$

In $\triangle PSR$, we have

$$\angle b + \angle c + 40^\circ = 180^\circ \text{ (Angle sum property)}$$

$$\Rightarrow 115^\circ + \angle c + 40^\circ = 180^\circ$$

$$\Rightarrow \angle c + 155^\circ = 180^\circ$$

$$\Rightarrow \angle c = 180^\circ - 155^\circ = 25^\circ$$

Hence, the required angles are $a = 65^\circ$, $b = 115^\circ$ and $c = 25^\circ$

3. Since I have three sides.

It is a triangle i.e. three-sided polygon.

Two angles are 15° and 60° .

$$\text{Third angle} = 180^\circ - (15^\circ + 60^\circ)$$

$$= 180^\circ - 75^\circ \text{ (Angle sum property)}$$

$$= 105^\circ$$

which is greater than 90° .

Hence, it is an obtuse triangle.

4. By the rule of Pythagoras Theorem,

Pythagoras theorem states that for any right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of square on the legs.

In the above figure RQ is the hypotenuse,

$$15^2 = 12^2 + a^2$$

$$225 = 144 + a^2$$

By transposing 144 from RHS to LHS it becomes -144

$$a^2 = 225 - 144$$

$$a^2 = 81$$

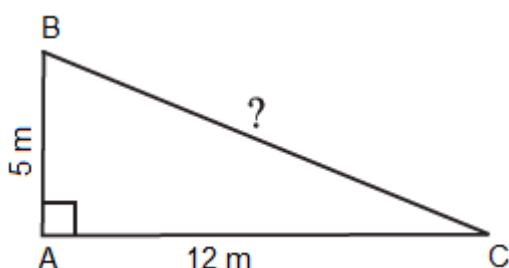
$$a = \sqrt{81}$$

$$a = 9 \text{ m}$$

Hence, the length of $a = 9 \text{ m}$.

5. Let ABC is the triangle and B is the point where tree is broken at the height 5 m from the ground.

Tree top touches the ground at a distance of $AC = 12 \text{ m}$ from the base of the tree,



By observing the figure we came to conclude that right angle triangle is formed at A.

From the rule of Pythagoras theorem,

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = 52 + 122$$

$$BC^2 = 25 + 144$$

$$BC^2 = 169$$

$$BC = \sqrt{169}$$

$$BC = 13 \text{ m}$$

Then, the original height of the tree = AB + BC

$$= 5 + 13$$

$$= 18 \text{ m}$$

Assertion and Reason Answers :

- 1) a) both Assertion and reason are correct and reason is correct explanation for Assertion
- 2) b) both Assertion and reason are correct but reason is not correct explanation for Assertion.